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13. ABSTRACT (*Maximum 200 words*)

The goal of this investigation was to demonstrate the existence of chaotic dynamic behavior in mechanical systems that contain rotating structures and/or fluid structure interactions. Progress was made in the study of the non-linear dynamics of rotating beams (simple models of helicopter rotor blades) and fluid flow induced experimental as well as analytical and numerical simulation. The research resulted in the award of one doctoral degree and offered support to visiting Professor M. Paidoussis, who built a test facility to study flow induced vibrations. In the study of rotating beams, we discovered that quasi-periodic vibrations were often a precursor to chaotic motions. Prof. Paidoussis introduced some strong non-linearities in the form of amplitude limiting constraints in his tube flow experiments. These constraints led to the transition of the original periodic flutter vibrations into chaotic flutter oscillations. The route to this chaos took place via period doubling bifurcations. One of the principal results of this study was the use of fractal mathematics in the experiments to prove that the dynamics of a continuous system could be modeled with a small number of ordinary differential equations. The techniques employed used ideas of fractal dimensions, Lyapunov exponents, auto-correlation, probability density functions, and bifurcation diagrams.

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CHAOTIC DYNAMICS IN ROTATING STRUCTURES AND FLUID-STRUCTURE PROBLEMS

Francis C. Moon, Principal Investigator

Professor and Director
Mechanical and Aerospace Engineering
Cornell University

Final Report to the Army Research Office
November 14, 1989

Significant progress has been made in the three years in the study of nonlinear structural dynamics and chaotic vibrations in two problem areas. These include the nonlinear dynamics of rotating beams and fluid flow induced experimental as well as analytical and numerical simulation. These two problems have illustrated the efficacy of the new ideas and techniques that have appeared in nonlinear dynamics including the use of fractal mathematics.

The support of the Army Research Office has resulted in the completion of a doctoral dissertation by Dr. Mohammad F. Golnaraghi and has resulted in five research papers and publications. This grant also made possible the visit last year of Professor Michael Païdoussis of McGill University, who helped design and build a test facility to study flow induced vibrations.

In addition to these reports and papers, the principal investigator has given over 30 invited lectures in the subject of nonlinear vibrations including a presentation at the International Congress of Theoretical and Applied Mechanics at Grenoble, France during August 1988.

Chaotic Dynamics in Rotating Structures

Chaotic dynamics has added a new dimension to nonlinear dynamics and may provide an understanding of deterministic sources of noise in mechanical systems. Chaos in dynamics systems arises from nonlinear effects. In our study of rotating beams, we looked at the role of kinematic nonlinearities such as centripetal and Coriolis acceleration effects when a structure vibrates in a rotating reference frame. Such problems are important in the mechanics of technical objects such as helicopter blades as well as certain robotic devices. Most studies of chaotic dynamics have focused on single degree of freedom problems with stiffness nonlinearities. In our study we looked at a two degree of freedom problem with kinematic nonlinearities. In addition the reference frame had a periodic but non-steady rotating rate meant to simulate a robotic type device such as a polar coordinate robotic manipulator with a flexible arm (Figure 1).

The primary result of this study was that quasi-periodic vibrations were often a precursor to chaotic motions. Quasi-periodic motions are vibrations with two or more incommensurate frequencies. While they are not periodic, they can be studied analytically and a comparison between theory and analysis was good.

Flow-Induced Chaotic Vibrations

This work was done with Professor Michael Païdoussis of McGill University. A test facility was built to study chaotic vibrations resulting from fluid flow through an elastic tube. Classical studies of this problem have shown that period flutter oscillations are excited when a fluid flowing through an elastic tube or pipe exceeds a critical velocity. In our study, however, we introduced some strong nonlinearities in the form of amplitude limiting constraints (Figure 2). These constraints led to the transition of the original periodic flutter vibrations into chaotic flutter oscillations. The route to this chaos took place via period doubling bifurcations.

One of the principal results of this study was the use of fractal mathematics in the experiments to prove that the dynamics in this continuous system could be modelled with a small number of ordinary differential equations. This mathematical model consisted of two coupled second order oscillators with cubic nonlinearities. This problem was a successful demonstration of the new methods of experimental and numerical analysis of dynamical systems as described in the Principal Investigator's new book **Chaotic Vibrations** (J. Wiley & Sons, 1987). The techniques employed used ideas of fractal dimensions, Lyapunov exponents, auto-correlation, probability density functions and bifurcation diagrams to analyze this problem.

It should also be noted here that the presence of the Mathematical Sciences Institute at Cornell, sponsored by the Army, was of great value to this project through the use of consultation with MSI-supported faculty and their help in interpreting experimental and numerical dynamical data.

Francis C. Moon

Publications Supported by ARO

1. Golnaraghi, M.F. and F.C. Moon, *Chaotic Dynamics of a Nonlinear Positioning Device with Feedback Control*, submitted to the ASME Journal of Dynamics Systems, Measurement and Control.

2. Golnaraghi, M.F. and F.C. Moon, *Chaotic Dynamic and Control of a Mechanical Positioning Device*, Ninth Symposium on Engineering applications of Mechanics, London, Ontario, Canada, May 29-31, 1988.

3. Golnaraghi, M.F., F.C. Moon and R.H. Rand, *Resonance in a High Speed Flexible-Arm Robot*, to be published in The International Journal of Dynamics and Stability of Systems **4**, (1989).

4. Golnaraghi, Mohammad Farid, *Chaotic Dynamics and Control of Nonlinear and Flexible-Arm Robotic Devices*, Ph.D. Dissertation presented to the Graduate School of Cornell University, 1988.

5. Païdoussis, M.P. and F.C. Moon, *Nonlinear and Chaotic Fluidelastic Vibrations of a Flexible Pipe Conveying Fluid*, J. Fluids and Structures **2** (1988) 567-591.

6. Païdoussis, M.P., Guangxuan Li and F.C. Moon, *Chaotic Oscillations of the Autonomous System of a Constrained Pipe Conveying Fluid*, submitted to J. Sound and Vibration.

Reference [4]
Golnaraghi
Ph.D. Dissertation

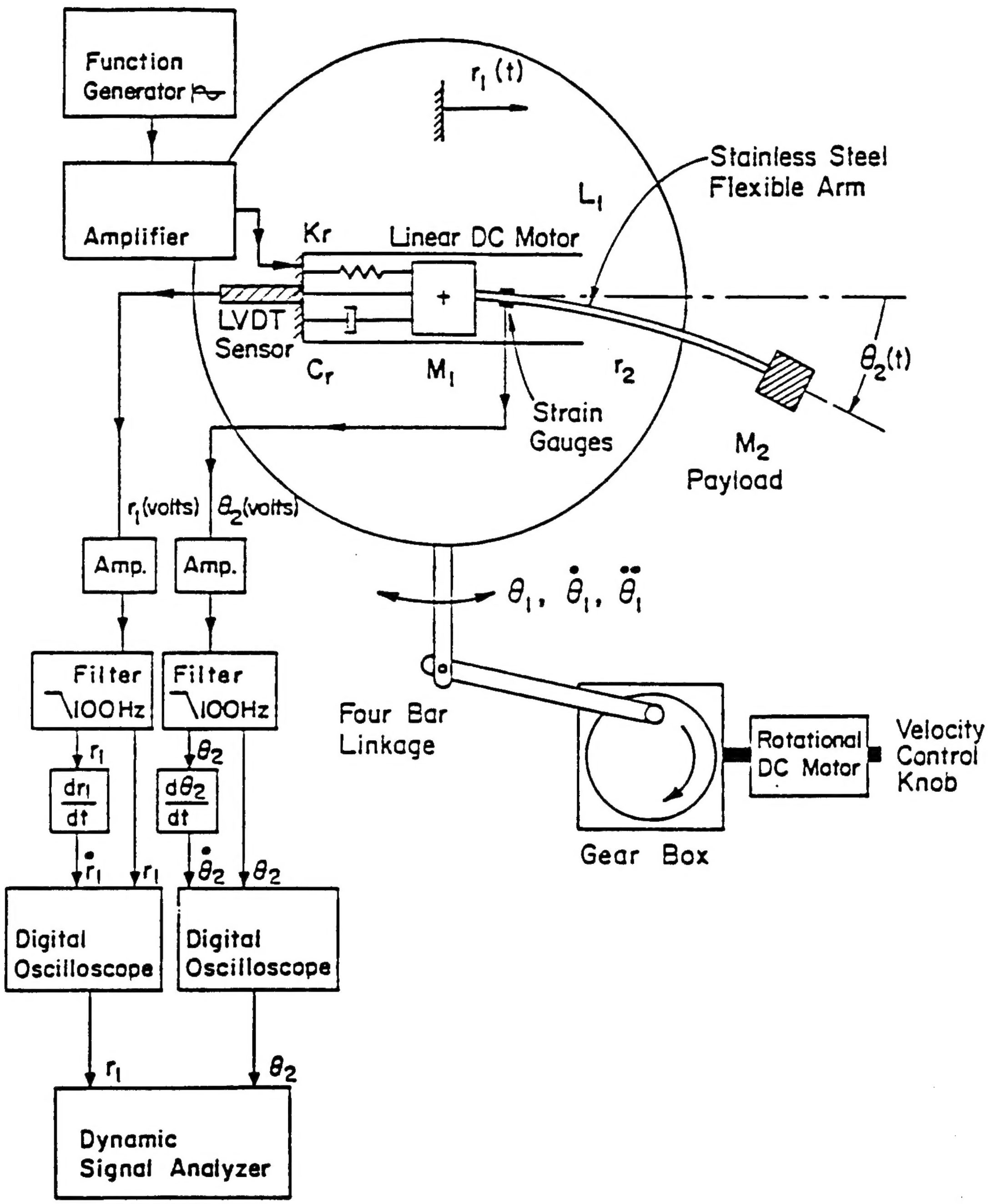

Fig. II.1.2: Experimental setup of the two degree of freedom manipulator.

Reference [4]
Golnaraghi
Ph.D. Dissertation

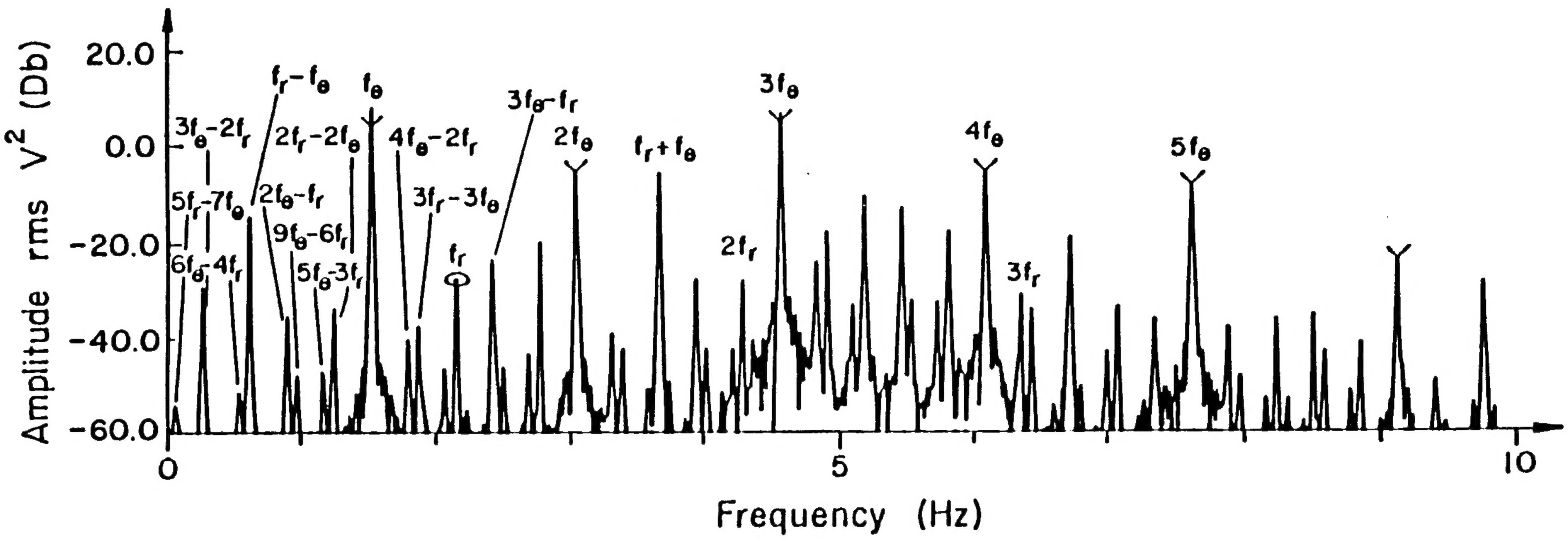

Fig. II.1.6: The experimental frequency spectrum of the rotational signal θ_2 showing the combination resonances as a function of $f_r = 2.15$ and $f_\theta = 1.522$ Hz.

Reference [5]
Moon, Paidoussis

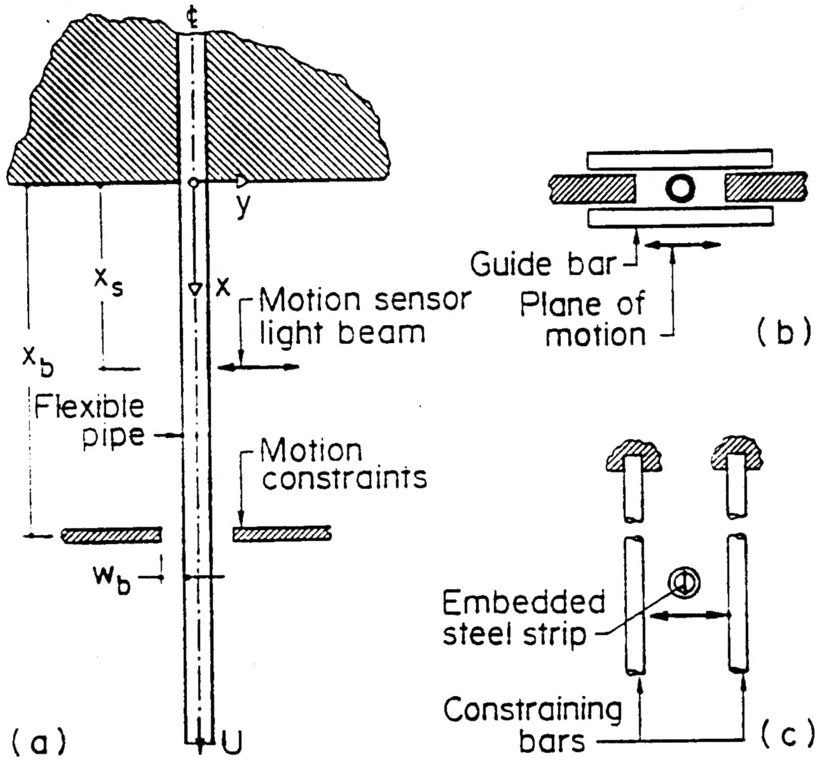

Figure 1. (a) Schematic of the experimental system; (b) scheme of achieving planar motions by guide-bars; (c) refined scheme for planar motions, with steel strip embedded in the pipe, also showing motion constraining bars.

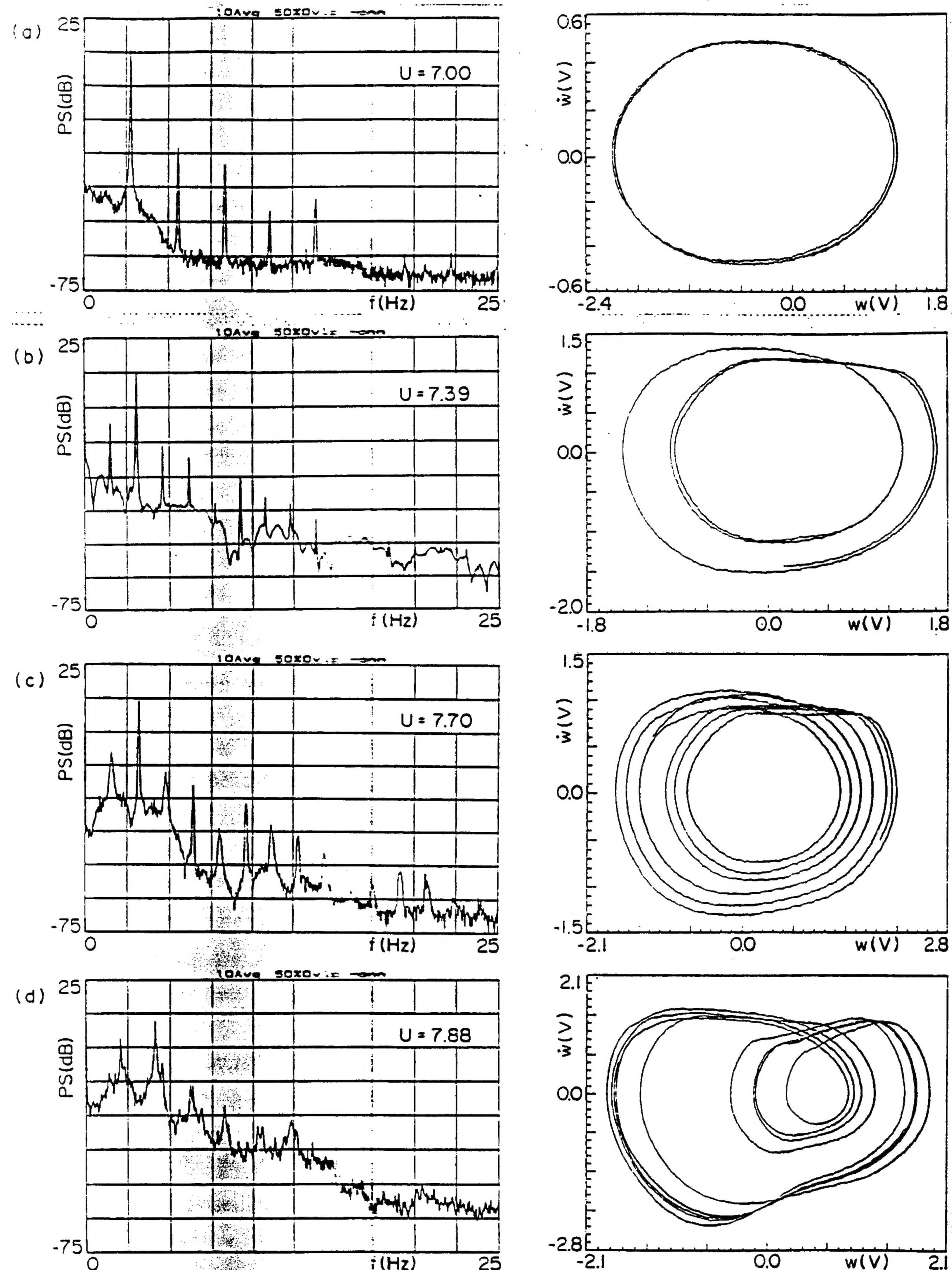

Figure 4. Vibration spectra and phase portraits for a pipe conveying water, for the flow velocities indicated (in m/s): recent set of experiments with the motion-constraint bars fitted with soft leaf-springs. For pipe and constraint parameters, see Tables 1 and 2.

Chaotic Dynamics of a Nonlinear Positioning Device With Feedback Control

by

M.F. Golnaraghi*
F.C. Moon+

October 1988

Submitted to: The ASME Journal of Dynamics Systems, Measurement and Control.

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+ Director and Professor, Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, New York, USA

ABSTRACT

In this paper the dynamics of a one degree of freedom mechanical positioning device with nonlinear stiffness, and to linear feedback, is described. Experimental and numerical evidences show that chaotic dynamics can result due to an increase in control gain or shuttle frequency. Adding sufficient damping through velocity feedback gain, however, eliminated the chaos. In some chaotic motions, a one dimensional delay map was obtained from the experimental time samples. The map was modeled by a piecewise linear function, which suggested that the third order system might be modeled by a lower dimensional equation.

Chaotic Dynamics and Control of a Mechanical Positioning Device

by

M.F. Golnaraghi+
F.C.Moon#

February 1988

Department of Theoretical and Applied Mechanics
Cornell University
Ithaca, New York 14853
U.S.A.

Ninth Symposium on Engineering Applications of Mechanics
London, Ontario, Canada, May 29-31, 1988

+ Graduate Research Assistant
# Director and Professor, Sibley School of Mechanical and Aerospace Engineering, Cornell University

ABSTRACT

The suppression of chaotic dynamics in a one degree of freedom mechanical positioning device with nonlinear stiffness, subject to linear feedback control, is the focus of this paper. Chaotic oscillations are random-- like vibrations appearing in a nonlinear dynamical system, even though the system is completely deterministic [1].

Experimental and numerical results provide evidence for the existence of chaotic oscillations in the output as the forcing frequency and or proportional control gain is increased. The chaotic motions are quenched, however, upon adding the appropriate rate of velocity feedback control. The feedback control gains are obtained based on pole placement algorithm.

The state space equations (i.e. equations of motion) of the system form three first order ordinary differential equations with a nonlinear stiffness. Computer simulation of the equations of motion yield qualitatively the same behavior as the physical system.

Thus, chaotic oscillations limit the upper frequency range of operation in a servomotor positioning device, and are the principle nonlinear phenomena observed in the system. The operating range of the device is increased, however, and the chaotic motions at high speeds are eliminated using velocity feedback. This work has been extended to investigate the nonlinear behavior of multi-degree of freedom systems such as space structures and high-speed robotic devices.

Resonance in a High Speed Flexible-Arm Robot

by

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R.H. Rand#

October 1988

Accepted to The International Journal of Dynamics and Stability of Systems, Volume 4, Number 4, Fall 1989.

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# Professor, Department of Theoretical and Applied Mechanics, Cornell University, Ithacas, New York, USA

ABSTRACT

In this paper we consider the effects of nonlinearities due to Coriolis and centripetal forces on the motion of a multi-degree of freedom high speed flexible-arm robot. We perform experimental investigations on a robot arm as well as analytical investigations on a mathematical model of the experimental apparatus. The two variable expansion perturbation method is used to describe the motions at internal and primary resonances. The perturbation solutions show the existence of a jump phenomenon and "saturation" when both forced resonance as well as when 2:1 internal resonance occur. This phenomenon is also observed in the experiments. Hence, proving the accuracy of the perturbation solution.

CHAOTIC DYNAMICS AND CONTROL OF NONLINEAR AND FLEXIBLE ARM ROBOTIC DEVICES

Mohammad Farid Golnaraghi, Ph.D.

Cornell University, 1988

Dynamical behavior and control of high speed robotic type mechanisms is the main focus of this dissertation with the major emphasis on the existence of chaotic oscillations associated with stiffness, Coriolis, inertia, and centripetal nonlinearities.

The work is divided into two catagories. In the first part the behavior of a one degree of freedom mechanical positioning device with nonlinear stiffness, subject to linear feedback, is described. In the second part, the effects of kinematic nonlinearities such as intertia, coriolis, and centripetal forces, on the motion of a multi-degree of freedom high speed flexible arm robot, is considered.

In the one degree of freedom problem, experimental and numerical results provided evidence that upon increasing the desired cycle frequency and or proportional control gain, the possibility of chaotic output, even when the input control was deterministic, would increase. Adding proper damping through velocity feedback gain, however, eliminated chaos and resulted in the positioning of the motion. In some cases, a one dimensional delay map was obtained from the experimental time samples. The map was modeled by a piecewise linear function, which suggested that the three dimensional system could be modeled by a lower dimensional equation.

The work in part two which considered a flexible arm robotic device undergoing planar motion, was an extension of studies on the one dimensional system discussed in part one. Numerical studies in this case, indicated the presence of chaotic behavior. The routes to chaos contained subharmonic bifurcations and quasi-periodic motions. The chaotic motions occured when the cyclic forcing frequency or amplitude were increased. Although the numerical model revealed similar qualitative behaviors as the experiment, chaotic solutions were not observed in the experiment due to the limitations of the apparatus. However, other nonlinear oscillations were observed in the experiments such as: subharmonic, quasi-periodic, and combination resonances. These phenomena were successfully predicted by analysis.

The two variable expansion perturbation scheme was used to describe the motions at internal and primary resonances. The perturbation solutions showed the existence of a jump phenomenon at the primary resonance when 2:1 internal resonance prevailed. This phenomenon was observed in the experiment, hence, proving the accuracy of the perturbation solution.

The chaotic vibrations which limited the upper frequency of operation were eliminated with feedback control.

NONLINEAR AND CHAOTIC FLUIDELASTIC VIBRATIONS OF A FLEXIBLE PIPE CONVEYING FLUID

by

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ABSTRACT

The dynamics of a cantilevered pipe conveying fluid have been investigated, both theoretically and experimentally, focusing on the nonlinear effects introduced by motion-limiting constraints. The experiments have indicated that, with increasing flow velocity, beyond the Hopf bifurcation (flutter) there are regions of period-doubling and chaos.

Fractal dimension calculations of the recorded vibration signals showed that finite-dimensional modelling of the system is possible. Accordingly, a two-degree-of-freedom, four-dimensional analytical model was implemented, with the aim of investigating the existence of chaotic vibrations in the parameter space of this autonomous system. Chaotic regions were indeed found to exist, with the aid of modern numerical techniques, involving the construction of bifurcation diagrams and the determination of Lyapunov exponents and power spectra. The analytical results are in qualitative agreement with experimental observations.

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2 Director, Department of Mechanical and Aerospace Engineering, Cornell University.

CHAOTIC OSCILLATIONS OF THE AUTONOMOUS SYSTEM OF A CONSTRAINED PIPE CONVEYING FLUID

by

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ABSTRACT

Experiments have shown that a cantilevered pipe conveying fluid, the limit-cycle motions of which (beyond the Hopf bifurcation) interact with motion-limiting nonlinear constraints, exhibits regions of chaotic motions. This paper examines the planar dynamics of the flexible pipe system theoretically by means of a two-degree-of-freedom (four-dimensional) analytical model, exploring the existence of chaotic oscillations in the parameter space of this autonomous system. Calculations of the Lyapunov exponents, phase portraits of the oscillation, bifurcation diagrams and Poincaré maps establish definitively the existence of chaotic motions. The route to chaos is shown to be via period-doubling bifurcations. The effect of some key parameters on the chaotic regions is investigated.

[1]Currently Visiting Scientist, Department of Theoretical and Applied Mechanics, Cornell University

Francis C. Moon

1988 Lectures and Seminars

21 January	Report to the Senate	New York City	"Magnetic Levitation"
14 February	AAAS Meeting	Boston, MA	"Chaotic Dynamics"
17 February	Rochester Polytechnic Institute	Rochester, NY	"Chaotic Dynamics"
3 March	Carnegie-Mellon University	Pittsburgh, PA	"Chaotic Dynamics"
11 March	SUNY Buffalo	Buffalo,NY	"Magnetic Levitation"
21 March	Argonne National Laboratories	Argonne, IL	"Magnetic Levitation"
22 March	Notre Dame University	Notre Dame, IN	"Chaotic Dynamics"
23 March	Illinois Institute of Technology	Chicago, IL	"Chaotic Dynamics"
24 March	University of Illinois	Urbana, IL	"Chaotic Dynamics"
25 March	Purdue University	West Lafayette, IN	"Chaotic Dynamics"
11 April	University of Michigan	Ann Arbor, MI	"Chaotic Dynamics"
12 April	Michigan State University	East Lansing, MI	"Chaotic Dynamics"
14 April	University of Wisconsin	Madison, WI	"Chaotic Dynamics"
15 April	University of Minnesota	Minneapolis, MN	"Chaotic Dynamics"
18 April	SUNY Buffalo	Buffalo, NY	"Magnetic Levitation"
28 April	National Research Council	Washington, DC	"Chaotic Dynamics"
1 June	High Speed Rail Asso. Meeting	Washington, DC	"Magnetic Levitation"
2 June	Virginia Polytechnic Institute	Blacksburg, VA	"Chaotic Dynamics"
21 August	IUTAM - International Congress	Grenoble France	"Chaotic Dynamics"
26 September	EURO-MECH Chaos Conference	Wuppertal, FRG	"Chaotic Dynamics"
19 October	Technische Universität	Wien	"Chaotic Dynamics"
22 October	Technical University	Budapest	"Chaotic Dynamics"
27 October	Technische Universität	Braunschweig	"Chaotic Dynamics"
31 October	Technische Universität	Hannover	"Chaotic Dynamics"
1 November	Technische Hochschule Darmstadt	Darmstadt	"Chaotic Dynamics"
18 November	Universität Wurzburg Department of Mathematics	Wurzburg	"Chaotic Dynamics"
26 November	Royal Academy of Technology	Stockholm	"Chaotic Dynamics"
6 December	Technische Universität Karlsruhe	Karlsruhe	"Chaotic Dynamics"
8 December	Technische Universität München	München	"Chaotic Dynamics"
13 December	Universität Bremen Dept. of Physics	Bremen	"Chaotic Dynamics"
14 December	Technische Universität Hamburg-Harburg	Hamburg	"Chaotic Dynamics"
15 December	Technische Universität Berlin	Berlin	"Chaotic Dynamics"
21 December	Swiss Federal Institue of Technology ETH-Zurich ---	Zurich	"Chaotic Dynamics" and "Superconducting Bearings"